Bunch Compressor for a Muon Collider

V.I. Balbekov, N.V. Mokhov
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

April 25, 2003

Abstract

Production of a short high-intensity bunch for a $\mu^+\mu^-$collider is considered. A system proposed includes a 24-GeV proton driver, jet mercury target in a 20-T solenoid, 30-m phase rotation-drift-decay channel, and 72-m bunch compressor based on a ring cooler with an intensive emittance exchange. A 36-MHz accelerating system with a gradient of 6.4 MV/m is used both in the phase rotation channel and ring cooler. The system produces a single muon bunch containing 0.11 muons per incident proton, with a longitudinal r.m.s. emittance of 2.5 cm, and transverse one of about 0.63 cm.

1 Introduction

The high physics potential of a $\mu^+\mu^-$collider can only be realized if all its components [1] achieve the required high performance. In an optimal regime, all muons generated should be collected in a single bunch, because for a given number of muons the luminosity is inversely proportional to the number of bunches. The bunch should be rather short because the required strong ionization cooling of muons can be provided only by using a high-voltage (small wave length) accelerating system. For example, a 805-MHz RF system with the accelerating gradient of 36 MeV/m, considered in Ref. [1], allows to accept and cool a bunch with a length of about $\pm$ 7.5 cm and energy spread of $\pm$ 40 MeV, i.e. with a total longitudinal emittance of about 3 cm. A 201-MHz RF system with a gradient of 15 MeV/m [2, 3], can accept a bunch of $\pm$ 30 cm $\times \pm$ 25 MeV with a total emittance of about 7 cm.

Unfortunately, muons generated in pion and kaon decays populate a very diffuse longitudinal phase space. Therefore, a simple capture of muons into a 201-MHz cooling channel could provide only a bunch containing about 0.01 muons per proton, that is 10-20 times less than required. A phase rotation linac was proposed to solve this problem [1]. Its primary goal is to decrease the energy spread of a muon beam to a level acceptable for a cooling channel. The phase rotation by an induction linac for a neutrino factory was considered in detail in Ref. [2, 3]. Another approach based on a variable frequency 300-to-180 MHz linac was proposed in Ref. [4]. However, the phase rotation does not raise density in a longitudinal phase space, and a number of particles in a muon beam grows only by an increase of a number of bunches. Such a way is quite acceptable for a neutrino factory but not optimal for a collider. Extremely long muon beams (40-60 m) which can be obtained by the above methods are unsuitable for a short (30-40 m) ring coolers which have been recently proposed and developed [5, 6]. Therefore we are reviving in this paper the earlier idea of using for phase rotation a medium frequency accelerating system (30-60 MHz) [1].

2 Phase Rotation via Medium Frequency Acceleration

The bunch compressor would include the following parts:

1. A proton driver supplying a target with a 24-GeV proton bunch of the r.m.s. radius 0.21 cm and length 90 cm.

2. A mercury jet target 0.5-cm radius and 2-m effective length. The target is placed in a solenoid with the axial field decaying adiabatically from 20 to 4.4 T.

3. A 20-m phase rotation–decay channel with a 36.37-MHz RF system providing an accelerating gradient of 6.37 MeV/m. The channel starts at 4.1 m from the target center. The solenoid field decaying adiabatically from 4.4 to 1.75 T is used for transverse focusing.

4. A drift–decay channel that is a 10-m long solenoid with a radius of 26 cm with the axial field of 1.75 T.

5. A bunch compressor, i.e. a 8-period ring cooler with a circumference of 72.29 m. It is based on principles described in Ref. [5], but with different parameters adjusted for the fastest longitudinal cooling. Each period includes two combined-function bending magnets, short solenoid with a wedge absorber, and a long solenoid where the main absorber and 36.37-MHz RF cavities are installed. The long solenoid field is only 1.75 T because a strong transverse cooling is not a goal of this stage. There is a field flip of about $\pm$ 2.35 T in the short solenoid. Note that emittance exchange only (without cooling) does not crack the problem because it simply substitutes the time-energy correlation for time-transverse coordinate one, with an unacceptable blow-up of the transverse emittance.

Only 36.37-MHz cavities with a gradient of 6.37 MeV/m are used everywhere in the system. It distinguishes the current consideration from the previous ones, where a set of 30-60 MHz or 30-90 MHz cavities was proposed as two options for a phase-rotation channel [1]. However, it turns out that addition of higher harmonics does not improve performances of the system. Using the same cavities for phase rotation and bunch compression simplifies substantially the system and is possible to the following reasons:
(a) Appreciable decrease of the energy spread in phase rotation is achievable for a beam fraction $\lambda/2$ long, i.e. about 5 m in our case.
(b) It is approximately what can be captured into the compressor at the same RF and a reasonable synchronous phase of 30°.

At these conditions, the frequency of the cavities should be a multiple of the revolution frequency in the ring cooler that is 3.637 MHz. Choosing the harmonic number, we sought to maximize the beam density in a longitudinal phase space, i.e. the following ratio:

The number of muons per bunch per proton / longitudinal emittance of bunch.

The following dependence of an accelerating gradient on frequency was taken at the above optimization:

$$V' \text{ (MV/m)} = 1.06\sqrt{F \text{ (MHz)}}$$

It was found that the 8th, 9th, and 10th harmonics give the best and almost identical results. Our preference was given to the harmonic number $h = 10$ and corresponding frequency 36.37 MHz. Such a system is considered in detail below.

3 Target and Primary Capture

The system starts with a proton beam impinging on a thick target sitting in a high-field solenoid (20 T, about 1 m long, aperture radius R_a=7.5 cm), followed by a 4.1-m long matching section where the field decays to 4.4 T. Optimization of beam, target and solenoid parameters was done over the years with the MARS14 code [7] for a $\mu^+\mu^-$ collider and a neutrino factory (see, *e.g.*, Ref. [1, 8]). The optimized configuration for the Study-2 [3], designed for a 1 MW proton beam of 24 GeV energy (upgradable to 4 MW), is shown in Fig. 1. The beam intensity is 1.7×10^{13} ppb $\times\, 6 \times 2.5$ Hz $=$ 2.55×10^{14} p/s, resulting in 5.1×10^{21} p/yr at 2×10^7 s/yr.

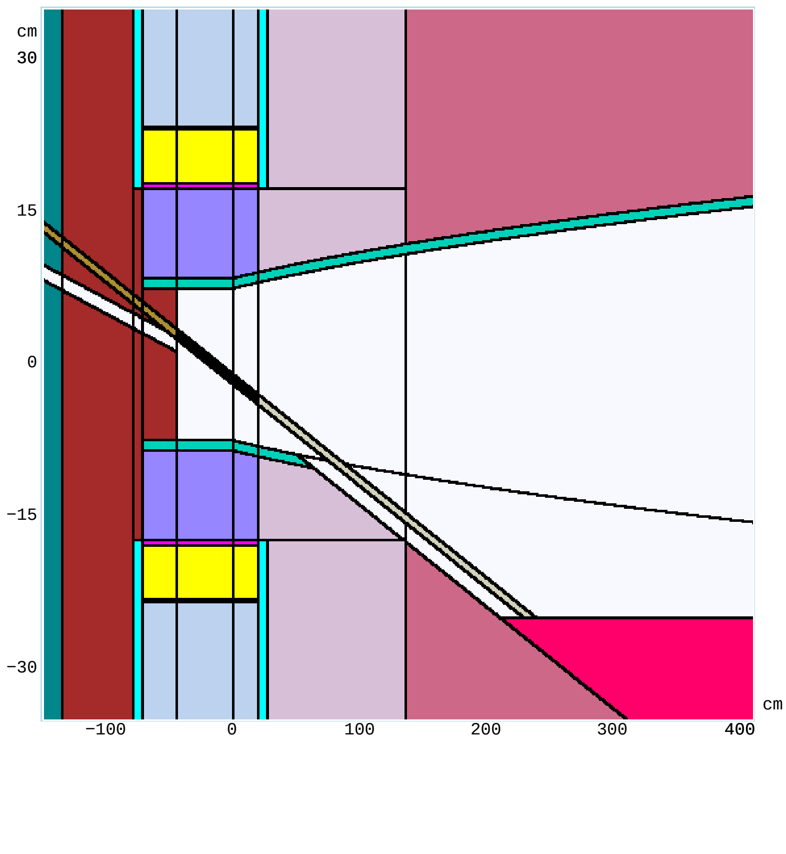

Figure 1: A fragment of the MARS model of target/capture system with tilted proton beam, mercury jet and mercury pool shown.

A proton beam (σ_x=σ_y=1.5 mm, σ_z=3 ns, 67 mrad) interacts with a 5 mm radius mercury jet tilted by 100 mrad. The beam and jet axes cross the solenoid axis at z = 0 as is shown in Fig. 1. Timing starts up when the center of the proton bunch crosses the z = -45 cm plane. The jet is ejected from the nozzle at z = -60 cm, and hits a mercury pool at z = 220 cm, x = -25 cm. The model was optimized for -2 $< z <$ 4.1 m and r$<$ 1.8 m. With such a beam-jet crossing, about 97% of protons have a probability to interact with target material, generating pions and resulting in significant energy deposition in material that can at some conditions destroy solid or liquid target. A 8-cm wide mercury pool is a core of a sophisticated spent beam absorber. A corresponding 2-mm beryllium window at z = 410 cm withstands beam-induced heating (with appropriate cooling), but its lifetime is an issue because the absorbed dose in its center reaches tens of GGy/yr.

A longitudinal phase space of negative pions and muons just after the target station (z = 410 cm) is shown in Fig. 2 (left). The energy distribution is very wide, and the yield peaks at the total energy of 200 - 300 MeV as seen from Fig. 2 (right). A transverse phase space (Fig. 3) has a rather unusual form because the beam stays in the 4.4-T magnetic field and has a systematic rotation counter-clockwise.

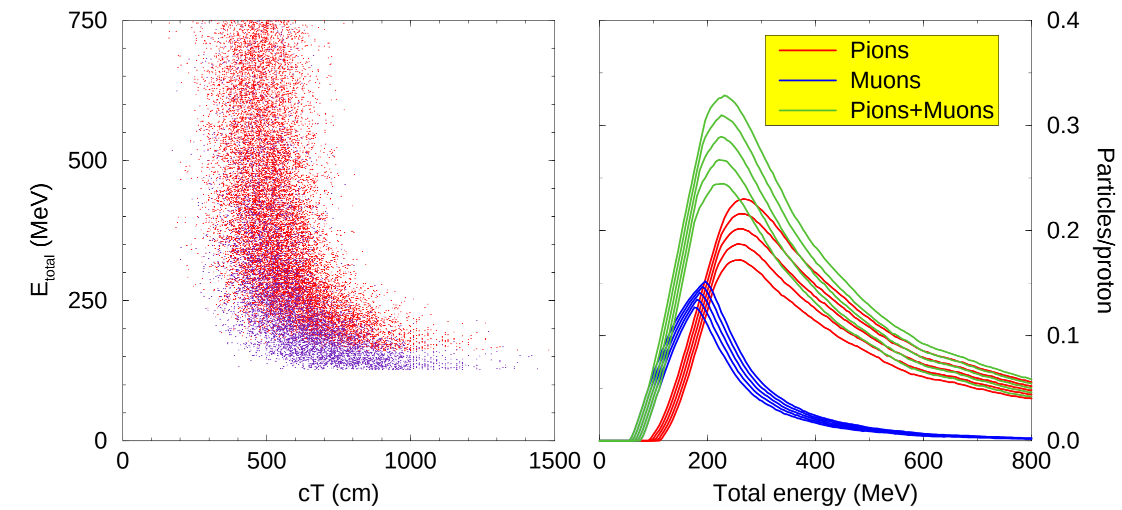

Figure 2: Longitudinal phase space of π^- and μ^- at z = 410 cm (left) and their numbers in five energy bands (100, 110, 120, 130 and 140 MeV from bottom to top) *vs* position of the interval center (right). Red points/lines – pions, blue – muons.

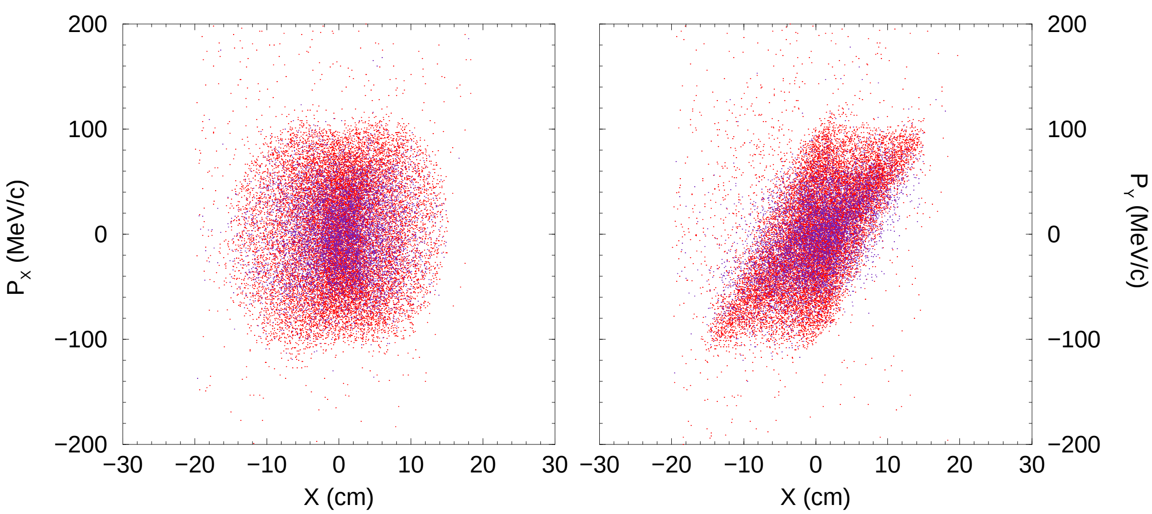

Figure 3: Transverse phase space of π^- and μ^- at z = 410 cm in horizontal (left) and vertical (right) planes. Red symbols – pions, blue – muons.

4 Phase Rotation - Drift - Decay Channel

A 30-m channel starts at 4.1 m from the target center. The axial 1.75-T magnetic solenoidal field in a 26-cm aperture is used for transverse focusing everywhere except for the first five meters where the field smoothly decreases from 4.4 to 1.75 T. Twenty 36.37-MHz cavities - spaced by 1 m - are installed from the channel entrance. The cavities are approximated by thin slots with a maximal voltage of $V_0 = 6.37$ MV. The solenoid coils have to be designed along with the cavities; below the axial field is described analytically. The voltage of the very first cavity is $V(t) = V_0 \sin 2\pi F(t - t_0)$, where time t is measured as described in previous section, $ct_0 = 700$ cm. Other cavities are tuned by the reference particle that is muon with a total energy of 800 MeV. Note that the results are slightly dependent on this choice. There are no cavities at the last ten meters of the channel.

The beam evolution at phase rotation is shown in Fig. 4 where a longitudinal phase space is presented at the end of the accelerating part at 20 m, and at the end of drift at 30 m. For comparison, the phase space after a simple 30-m drift without any RF is also shown to estimate efficiency of the phase rotation. One sees that the beam after the phase rotation is more concentrated. It is confirmed by Fig. 5 where a number of pions and muons in five energy bands is plotted as a function of the interval center, and Fig. 6 where the same is done for the 5-m fraction of the beam. Such a beam

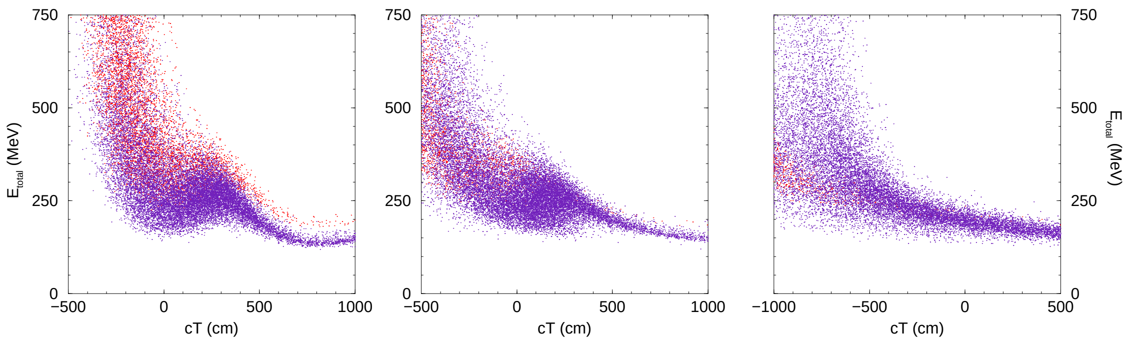

Figure 4: Longitudinal phase space of π^- (red) and μ^- (blue) after phase rotation (left), after additional drift (center), and after a simple 30-m drift at the solenoid without RF.

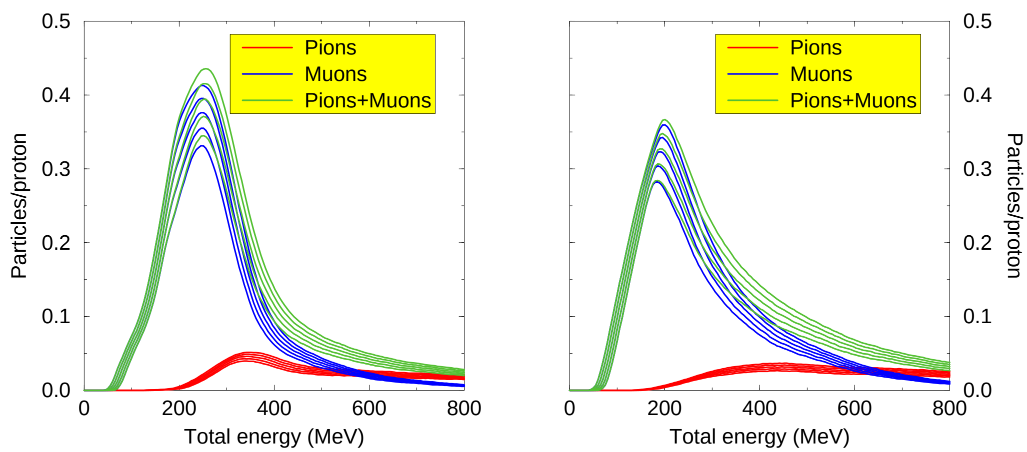

Figure 5: A number of π^- and μ^- their numbers in five energy bands (100, 110, 120, 130 and 140 MeV from bottom to top) *vs* position of the interval center after phase rotation and drift (left) and a simple 30-m drift (right). Red points/lines – pions, blue – muons.

fraction is approximately the effective length of the bucket in the ring compressor. A longitudinal position of the "captured" portion is optimized to hold as many particles as possible. It turns out that the phase rotation for decaying particles would increase the capture by a factor of about 1.6.

Transverse phase space after the phase rotation is shown in Fig. 7. The beam has rather unusual form because it is in a 1.75-T solenoidal magnetic field. After leaving the solenoid, it becomes approximately a Gaussian with $\sigma_{x,y} = 8.4$ cm and $\sigma_{p_{x,y}} = 22$ MeV/c, corresponding to a normalized transverse emittance of 1.75 cm.

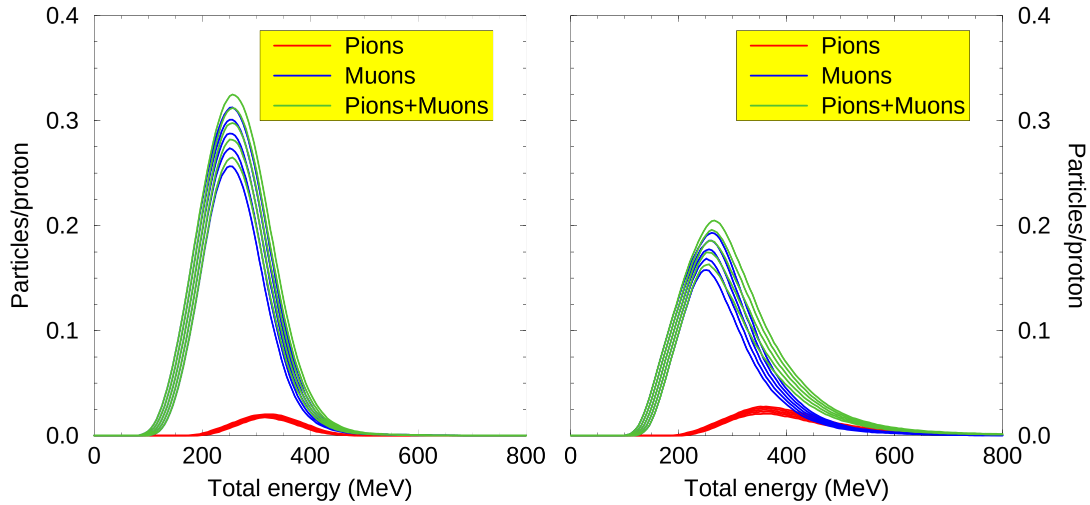

Figure 6: The same as Fig. 5 at collection of particles in a 5-m beam fraction.

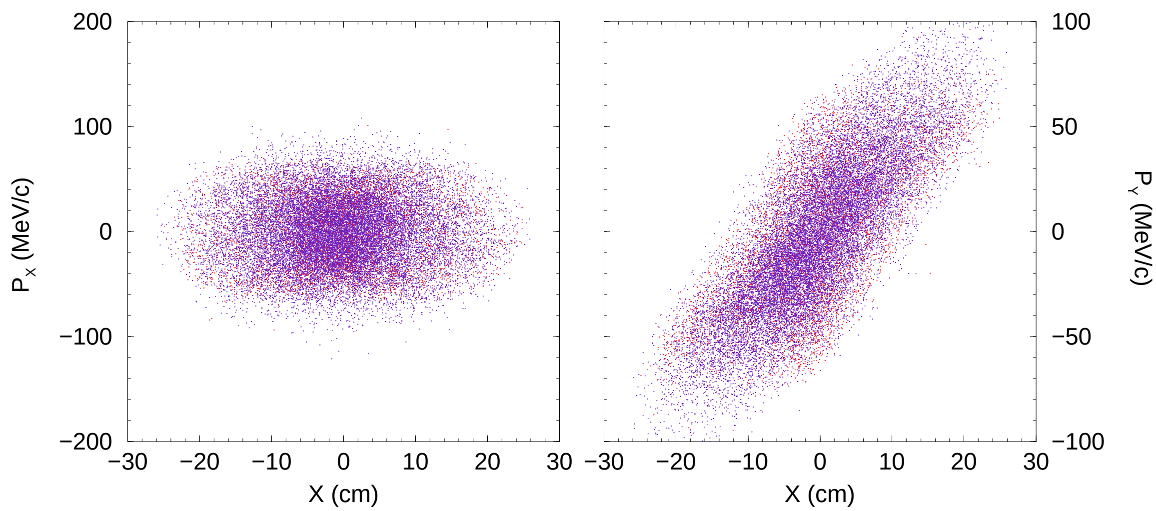

Figure 7: Transverse phase space of π^- and μ^- after phase rotation and drift in horizontal (left) and vertical (right) planes. Red symbols – pions, blue – muons.

5 Ring Cooler - Bunch Compressor

Further compression of the bunch is performed by a ring cooler based on the principles described in Ref. [5]. Similar use of a ring cooler was proposed in Ref. [9], however, a four-periodic ring considered there, had a poor performance, mostly because of an extreme large dispersion. Therefore, we follow here the octagonal scheme proposed in Ref. [10], with parameters optimized for the current consideration.

The layout and parameters of the Ring Cooler - Bunch Compressor are given in Fig. 8 and Table 1. The system includes of eight sectors, each of them consists of two bending magnets, long and short straight sections. Combined function dipole magnets with a field index of 0.5 are used to provide a 22.5° bend abd focusing.

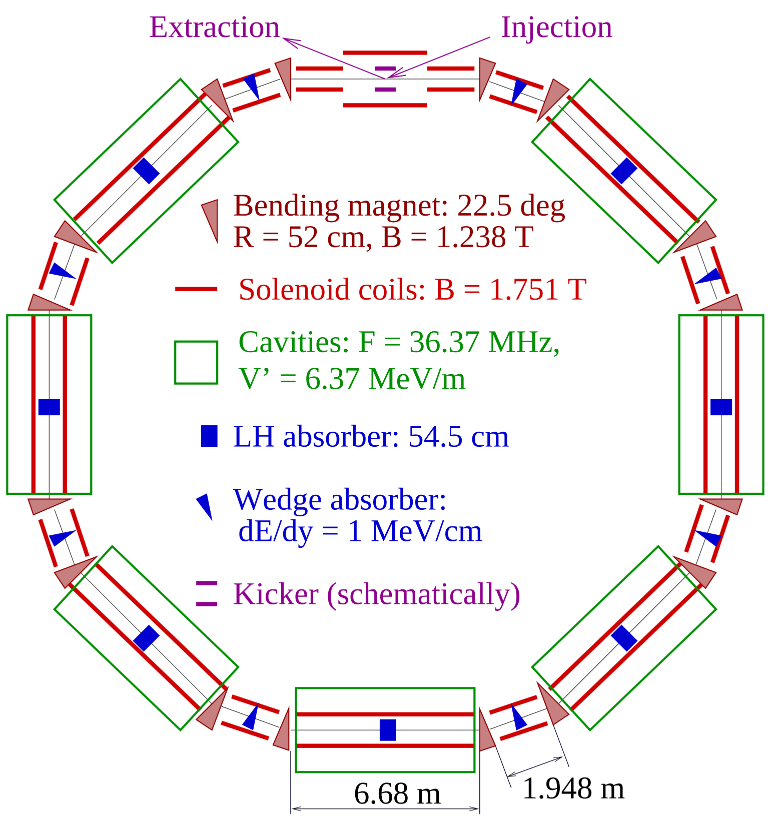

Figure 8: Layout of Ring Cooler - Bunch Compressor

The RF cavities and liquid hydrogen main absorbers are placed in long straight sections, which are the solenoids with a constant field $\pm$ 1.75 T of an alternating direction in sequent long sections. It provides the transverse focusing with a β-function of 73.5 cm (the same as bending magnets) at the total muon energy of 220 MeV. Because of so large β-function, a rather poor transverse cooling is possible. However, it is not the goal in the considered case, because almost all cooling is directed to the longitudinal degree of freedom by means of lithium-hydride wedge absorbers in the centers of the short straight sections. A schematic of such a section with solenoid coils is presented in Fig. 9, while Fig. 10 shows its axial magnetic field. Note that the neighboring bending magnets are treated as perfect magnetic mirrors in the field calculation, with the interference of dipole and solenoid fields neglected anywhere. Such a hard-edge approximation is rather crude for short high-aperture magnets, and the problem has to be considered further in a more realistic approximation.

Table 1: Parameters of Ring Cooler - Bunch Compressor

Circumference	72.291 m
Nominal energy (total)	220 MeV
Bending radius	52 cm
Bending field	1.2378 T
Normalized field gradient	0.5
Length of short SS	1.948 m
Length of long SS	6.68 m
Axial field of the long solenoid	1.7506 T
Revolution frequency	3.637 MHz
Accelerating frequency	36.37 MHz
Accelerating gradient	6.37 MeV/m
Synchronous phase	30°
LH_2 main absorber, length	54.5 cm
LiH wedge absorber, dE/dy	1 MeV/cm

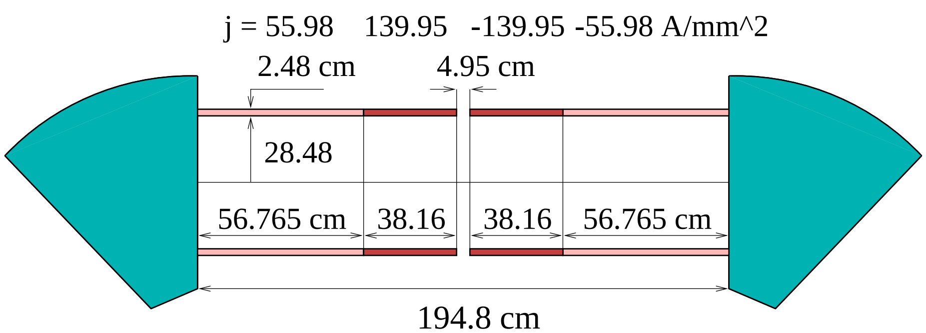

Figure 9: Layout of a short straight section.

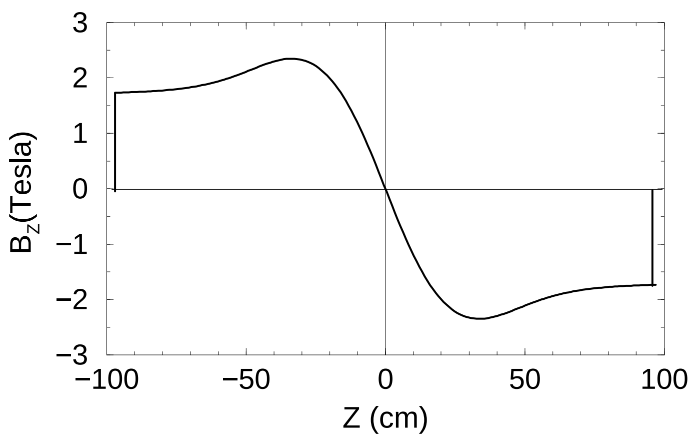

Figure 10: Axial magnetic field in the short straight section.

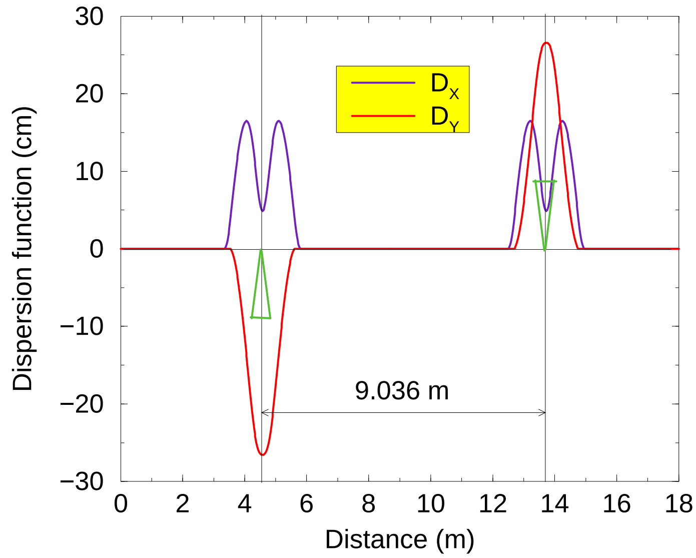

Figure 11: Dispersion function in a quarter of the ring.

The magnetic field of the short solenoid is chosen to provide zero dispersion in a long straight section because it improves a stability of a longitudinal motion. The corresponding dispersion function is shown in Fig. 11. Needless to say this is only a linear part, and some dispersion exists in long sections because of nonlinear additions. This effect worsens the longitudinal stability, and it weakens when the bending angle per period decreases that gives a preference to the octagonal compressor compared to the tetragonal one [10].

The β-functions in the ring is shown in Fig. 12 in dependence on beam energy and distance. The β-function at the absorber center can be described as $\beta^*(\mathrm{cm}) \simeq 0.38 \times p$ (MeV/c), except for several half-integer resonances. The effect of these resonances is actually negligible, mostly because of a short cooling time and suppression of large amplitudes by the dipole nonlinearities.

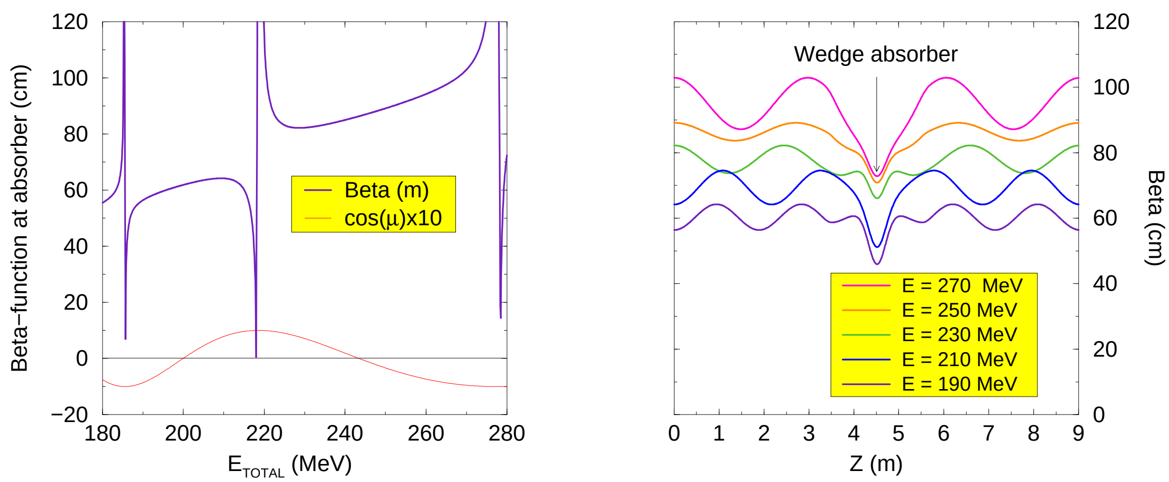

Figure 12: The β-function of the bunch compressor at the absorber center *vs* beam energy (left) and in dependence on distance at different energies (right).

Injection and extraction kicker is not considered in this paper and shown in Fig. 8) schenatically. Actually the accelerating cavities and absorber are placed in this region the way similar to the other long sections. With a lack of information on the cavities, we assume here that they are distributed uniformly in 2.67-m regions upstream and downstream of the absorber providing the average gradient of 6.37 MeV/m. With these assumptions, Figs. 13 and 14 show calculated behaviors of muon beam parameters and phase space in the bunch compressor. Only muons are considered assuming that pions do not survive in the ring and can't give birth to muons.

Evolution of the beam emittance and transmission is shown in Fig. 13. The muon bunch is formed over about two first turns by scraping of the uncaptured particles. The bucket shape becomes quite conventional after that, and the muons are lost almost exclusively due to decay, and emittance decreases by cooling. Because of a large β-function, the effective transverse cooling ceases after 4-5 turns, whereas the longitudinal cooling lasts for at least 13-14 turns.

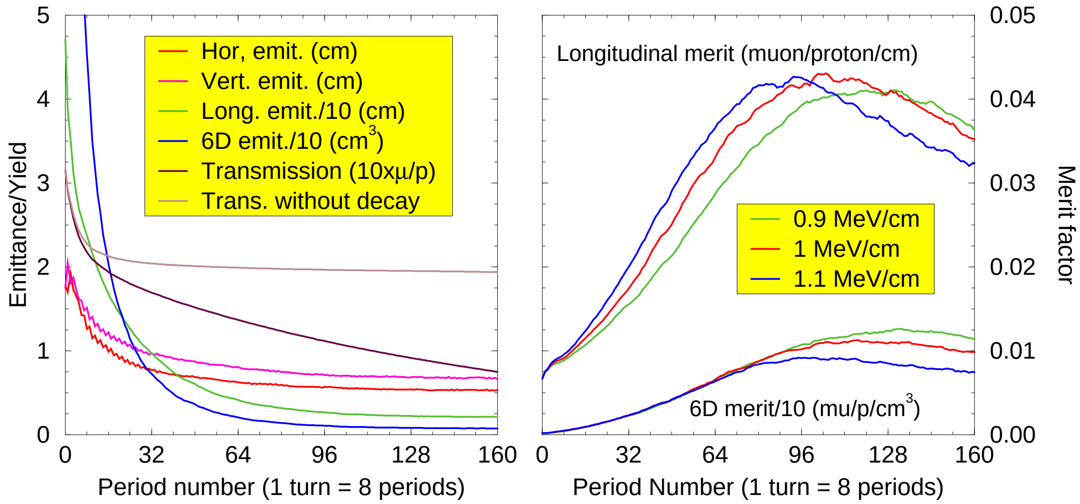

Figure 13: Evolution of muon beam parameters in the bunch compressor: emittance and transmission (left) and merit factors (right).

Fig. 13 shows also evolution of the longitudinal and 6-D merit factors. Contrary to a usual procedure, they are not normalized to the initial emittance, because the choice of an initial longitudinal phase volume is quite arbitrary, and this could hamper a comparison of different cases. Therefore, the following merit factor is used:

$$M.F. = \frac{\text{muon}/\text{proton}}{\text{emittance}}.$$

Three curves of each kind are obtained for different strengths of the wedge absorber. It is seen from the upper curves that the maximal longitudinal merit is achieved for the wedge strength of 1 MeV/cm after 13 turns. A stronger wedge allows for a faster growth of the merit at the beginning, but its maximal value is smaller because of earlier saturation. This is just opposite for a weaker wedge: slower and longer cooling. Note that the 6-D merit factor reaches its maximum for a stronger wedge absorber, and after longer cooling.

Evolution of the longitudinal phase space of the bunch at cooling is shown in Fig. 14. The initial window is rather arbitrary, however, the bunch is almost shaped after the first turn. This timme, there is a narrow "bottle neck" to the right where muons continue to run away. However, it becomes really closed after the second turn, and subsequent decrease of intensity is caused almost exclusively due to decay.

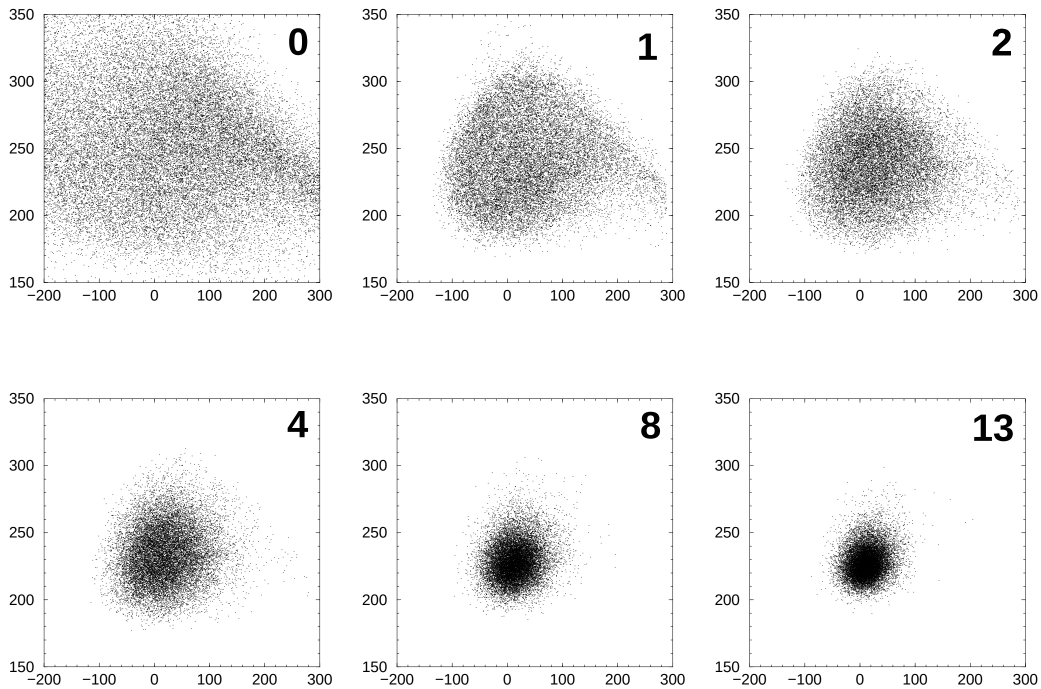

Figure 14: Longitudinal phase space in the compressor after: 0, 1, 2, 4, 8, 13 turns. Horizontal axis - *c* (cm), vertical axis - total energy (MeV).

6 Further Optimization

The nearest goal is the ultimate frequency choice and corresponding optimization of all the parameters. The beam characteristics obtained at different frequencies are given in Table 2. The higher frequency allows to get a smaller emittance, because of a higher accelerating gradient and faster cooling. However, the beam intensity is lower in the case, mostly because of a shorter bucket. Additionally, the phase rotation-drift channel becomes too short for higher frequencies to provide decay of all pions. Undoubtedly, the parameters would be significantly better if a higher accelerating gradient at a given frequency is achievable.

Table 2: Parameters of a cooled bunch after compression at different harmonic numbers.

RF harmonic number	7	8	9	10	11
Frequency (MHz)	25.46	29.10	32.74	36.37	40.01
Accelerating gradient (MV/m)	5.34	5.70	6.05	6.38	6.69
Number of turns	14	14	13	13	13
Number of muons/proton/bunch	0.13	0.12	0.12	0.11	.098
Long. phase density (muon/proton/cm)	.039	.040	.043	.043	.042
Horizontal emittance (cm)	0.60	0.61	0.58	0.56	0.54
Vertical emittance (cm)	0.76	0.78	0.74	0.71	0.68
Longitudinal emittance (cm)	3.3	3.0	2.7	2.5	2.4

7 Conclusion

Described system is capable to produce muon bunch with the characteristics which match the requirements of a subsequent cooling both for linear cooling channels (see.*e.g.*, [2, 3]) and ring coolers [5, 6].

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