# Heat Distribution in a Beryllium Foil Used in a High Gradient RF Cavity

by

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# 1. Introduction

In any scheme for a muon collider the preparation of the muons has to be done as soon as possible due to the finite lifetime of the muon [1]. Therefore the production, the capture, the cooling and the acceleration requires high gradient rf structures (generally well above 20-30 MV/m) in a frequency range of 0.03 to 1.0 GHz. In the cooling channel low energy muons ($\gamma\approx 2$, $\beta\approx 0.85$) travel through absorbing material and loose transverse and longitudinal momentum. In the accelerating rf structures only longitudinal momentum is replaced which results in a reduction in transverse phase space (transverse cooling). High gradient 805 MHz rf-structures are foreseen which operate at a gradient well above 30 MV/m and at a duty cycle mainly given by the filling time of the structure and the pulse repetition rate. The technical design of the structure is presented in [2]. One of the specialties of the structure is a 125 μm beryllium window, build into each iris, to increase the electric field strength on axis ($E_{peak} = E_{acc}$). Apart from the difficulty to incorporate the beryllium window in the manufacturing process, the Beryllium and Copper do have significantly different material properties, which makes rf operation difficult as well. The power dissipation in the beryllium foil and the subsequent heating is investigated in this paper.

# 2. The Geometry of the Cavity

The geometry of the cavity is shown in Figure 1 with three out of 12-16 cells which will later on be used for a full structure. The structure is designed to operate in $\pi/2$ mode. Every next but one cell is coupled through a coaxial type side coupled cell. The beryllium window prohibits electrical coupling through the iris.

| **Parameter** | **Unit** | **Beryllium** | **Copper** |
|---|---|---|---|
| **Melting Point** | **C°** | 1287 | 1085 |
| **Atomic weight** | | 9.01 | 63.5 |
| **Density ρ** | **g/cm$^3$** | 1.85 | 8.96 |
| **Thermal exp. α at 25 C°** | **$\times 10^6$ 1/K** | 11.3 | 16.5 |
| **Heat capacity (25 C°)** | **J/kg/K** | 1825.0 | 385 |
| **Heat cond.** | **W/m/K** | | |
| **25 C°** | | 200.0 | 401.0 |
| **77 K°** | | | 600.0 |
| **Electr. Resisit. ρ** | **$\times 10^{-8}$ Ωm** | | |
| **77 K°** | | 0.07 | 0.20 |
| **270 K°** | | 3.00 | 1.54 |
| **300 K°** | | 3.76 | 1.73 |
| **400 K°** | | 6.76 | 2.40 |

**Table 1:** Selected material properties of a beryllium and copper relevant for the power loss and heating calculations.

The electric field lines terminate on the beryllium foil and rf current flows in the surface layer of the beryllium window as well. Some material properties relevant for the calculations are summarized in Table 1.

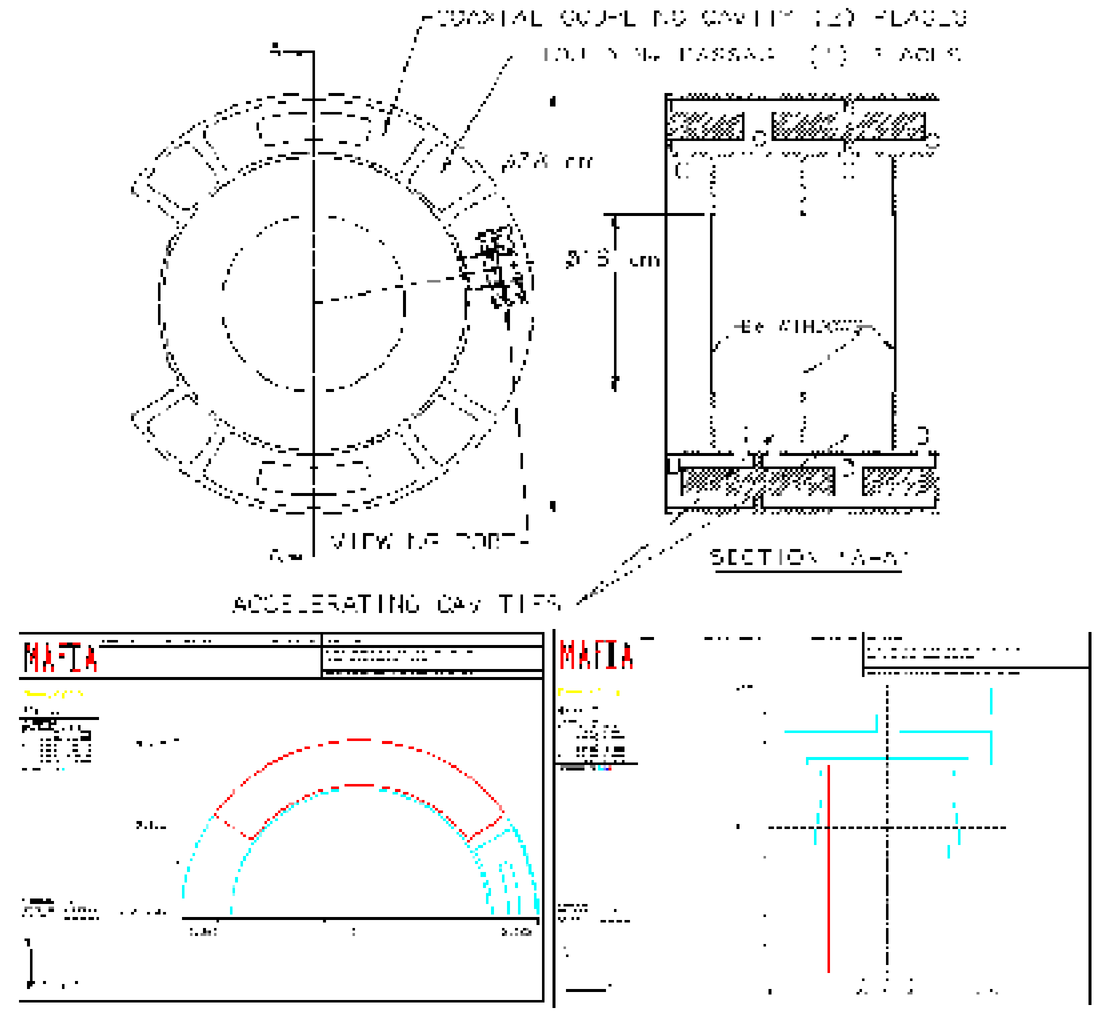



**Figure 1**: The shape of 2 cells of the accelerating structure is shown. Within the iris the 125 μm thick beryllium window can be seen. Every next but one cell is coupled through a coaxial type coupling cavity. The MAFIA model used for the calculation is shown in the two lower pictures, where the geometry of the coupling slot (left picture) and two coupled half cells (left picture) can be seen.

## 3. Estimates using the Pillbox

For a pillbox, the power dissipation can be calculated analytically. The dissipated power is simply given by:

(1) $$P_d = \frac{1}{2} \cdot \int_S R_w \cdot H^2 dS$$

For the pillbox the magnetic field is well known and given by a bessel-function,

(2) $$H_\theta = -\frac{E_z}{Z_0} \cdot J_1(k \cdot r); \qquad k = \frac{2.4}{R}; \qquad R = Cavity \quad Radius$$

and the integral over the surface can be devided into the integral over the outside walls with constant magnetic field and the integral over the two iris walls:

(3) $$\int_s \left| H^2 \right| ds = 2 \cdot \int_0^R J^2{}_1(k \cdot r) \cdot 2\pi r \cdot dr + 2\pi R \cdot l \cdot J^2{}_1(k \cdot R)$$

2x iris outside wall

The Ratio of the Power dissipated in the outside wall and the iris is therefore simply given by :

(4) $$\frac{P_{out}}{P_{iris}} = \frac{2\pi R \cdot l \cdot J^2{}_1(k \cdot R)}{2 \cdot \int_0^R J^2{}_1(k \cdot r) \cdot 2\pi r \cdot dr} \qquad with: \int_0^R J^2{}_1(k \cdot r) \cdot r \cdot dr = \frac{R^2}{2} J^2{}_1(k \cdot R)$$

which leaves a very simple formula for the ratio of the power dissipated in the outside wall $P_{out}$ to the power dissipated in the iris $P_{iris}$. It is the ratio of the cavity length to the cavity Radius:

(5) $$\frac{P_{out}}{P_{iris}} = \frac{l}{R}$$

For a given mode of operation, both the length l and the Radius R can be expressed as a function of wavelength, with the mode given in units of π :

(6) $$l = Mode \cdot \frac{1}{\pi} \cdot \frac{\lambda}{2}$$
$$R \equiv \frac{\lambda}{2.62}$$

For the mode of operation under consideration here (the π/4-mode) this turns out to be:

$$\text{(7)} \qquad \frac{P_{out}}{P_{iris}} = \frac{2.62}{4} = 0.65 \quad or \quad \frac{P_{iris}}{P_{tot}} = 0.6$$

This means that for a π/2 mode in a pillbox cavity 60 % of the power is dissipated in the iris(es).

A summary of the rf properties of the cavity (compare[2]) and some geometric parameters are presented in Table 2.

| **Parameter** | **unit** | |
|---|---|---|
| shunt impedance Rs | MΩ/m | 37 |
| Quality factor Q | | 20,000 |
| Peak Power PP | MW/m | 30 |
| Peak Power per cell | MW | 2.5 |
| filling time Tf | µsec | 3.9 |
| RF on time 3 x Tf | µsec | 12 |
| Repetition Frequency Frep | Hz | 15 |
| Average Power per cell | W | 440 |
| Power into the Beryllium window | W | 80 |
| Energy dumped in Beryll. per Pulse | J | 5.4 |
| Temp. rise per pulse ΔT/pulse in 2 δ | C° | 7.2 |
| Temp. rise per pulse ΔT/pulse | C° | 0.06 |
| **Geometric data for the cavity** | | |
| Beryllium window thickness | µm | 125 |
| Window radius | cm | 8 |

**Table 2:** RF parameters for the cavity using beryllium windows

Assuming that all the power would go through a surface given by the radius r = 0.7*R multiplied by the thickness of the window and transported over a length of 0.24*R, because most of the heat is dissipated close to the edge, the temperature rise can be estimated using:

$$\text{(8)} \qquad \Delta T = \frac{\Delta P \cdot L}{\lambda \cdot F} = \frac{\Delta P \cdot R/4}{\lambda \cdot 2\pi \cdot (0.7 \cdot R) \cdot d} = 180 C^{\circ}.$$

## 4. Calculation of the RF Power Loss and the Heat Distribution

The temperature increase per unit surface area is given by the power dissipated between radius 0 - r and has to be transported through a surface layer given by the thickness of the window ( D ) and the thickness of the layer Δr

$$\Delta P_d = \lambda \frac{2\pi \cdot r \cdot D}{\Delta r} \cdot \Delta T$$

**(9)**

$$\frac{\Delta T}{\Delta r} = \frac{1}{\lambda \cdot r \cdot D} \cdot \frac{1}{2} \int_r R_w \cdot H^2 r dr$$

The resistance of Beryllium in addition changes strongly with temperature, which is taken into account in the calculation. In Figure 3 the temperature gradient (dT) and the temperature (T) over the window is shown. The solid lines show the gradient and temperature including the dependence of the resistance on temperature. According to Table 1 the dependence is much stronger for beryllium than for copper

The temperature dependence and the total temperature gradient calculated so far for a pillbox cavity are in agreement with [3]. In addition the difference in shunt impedance between a pure pillbox cavity and the interleaved structure must be taken into account. The peak power required in a pillbox cavity to produce a certain gradient is only 72 % of the power required in the interleaved structure, which can easily be calculated using the previous formulas. The second picture in Figure 1 therefore shows a scaled curve using this value. Finally the temperature difference over the window turns out to be 191 deg if the cavity is operated at room temperature.

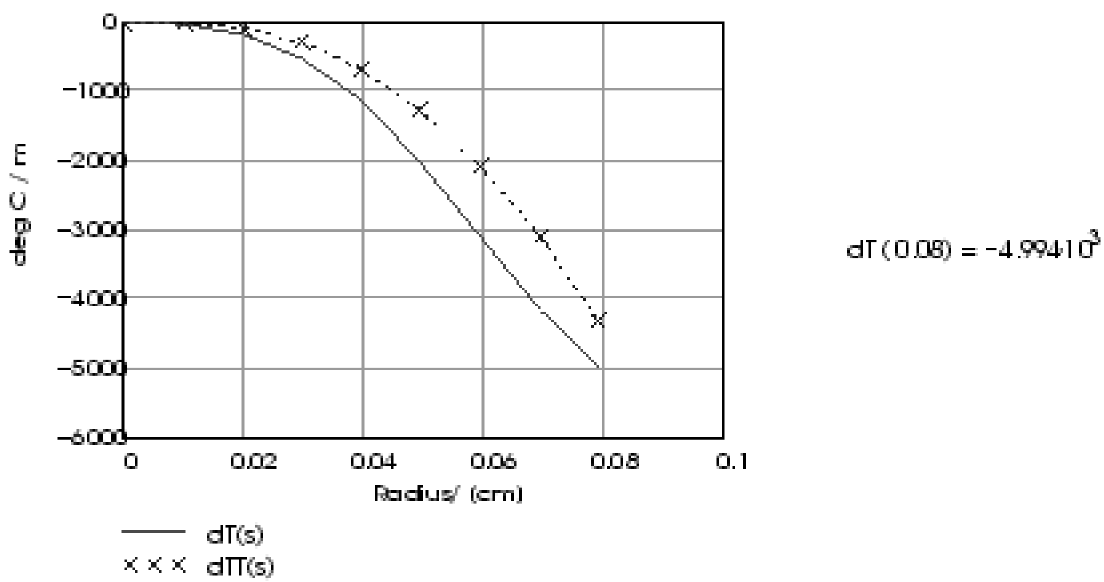
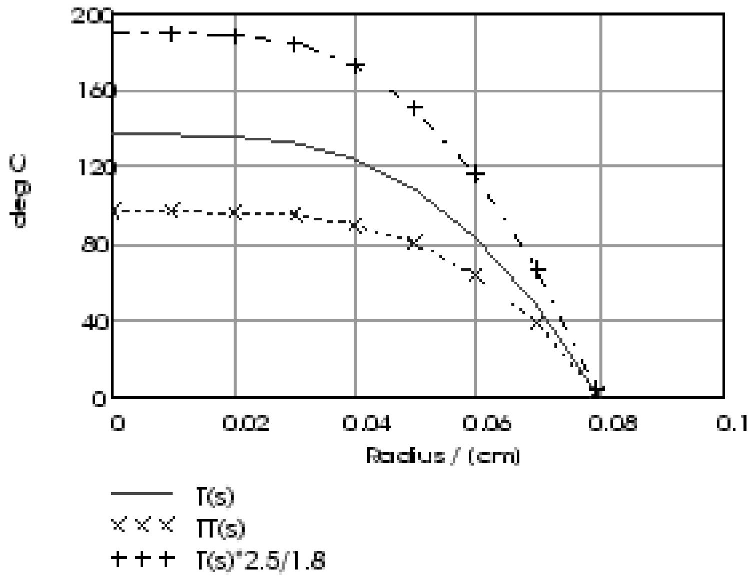



**Figure 2 :** The graph shows the temperature gradient over the window (Radius=0 middle of the iris). The dashed lines (dTT) is calculated not including the change in resistance with temperature.

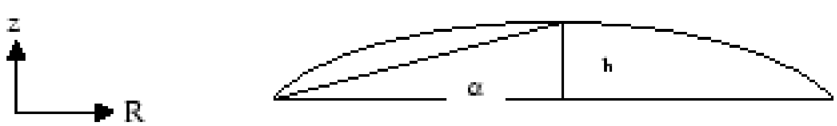



$$T(0)\frac{2.5}{1.8} = 191$$

Figure 3: The graph shows the temperature. The dashed line (TT) is calculated not including the change in resistance with temperature, while the continuous line includes the chnage. From the plot, one can see that this effect rises the temperature by 20 deg. In addition, the crossed (+) line includes the difference between the pillbox cavity and the actual cavity as shown in Figure 1, which has a lower shunt impedance plus the change in resistance. The maximum temperature rise to the center of the window is 191 dec C.

## 5. Mechanical deformation of the Beryllium window

Due to the temperature difference across the beryllium window, the window while being heated by rf power will deform itself. The amount of deformation can easily be estimated assuming the following geometry:




The window will bend to one side because of the thermal expansion of the disk

**Figure 4:** Simple geometry to estimate the geometric deformation of the window.

The total change in length can be calculated by integrating the temperature distribution according to:

**( 10 )**
$$L = 2\int_0^R 1+\alpha\cdot T(r)dr \qquad \Delta L = L - 2\cdot R = 2\cdot R\cdot(1-\cos(\alpha))$$

which gives an increase in length of 0.164 mm for a 16 cm diameter window. According to the geometry sketched in Figure 4 the movement of the window is:

$$h = \sqrt{R \cdot \Delta L} = \sqrt{80 * 0.141} = 3.4mm \quad (11)$$

From equation ( 11 ) one can see that the buckling height only decreases as the square root of the window expansion which will be of interest if one cools the cavity to liquid nitrogen temperature. The buckling described here will result in a shift of the resonant frequency of approximately 12 MHz.

## 6. Operation at Liquid Nitrogen Temperature

At liquid nitrogen temperatures the bulk properties of Beryllium changes drastically which improves the situation. Presently it is not clear, if full use can be made from these changes at low temperature and experiments are underway to determine those in a realistic environment. The essential parameters are listed in Table 3. The thermal conductivity was estimated using the Wiedemann-Franz law. The calculated value is 30% higher than the one given in the Table 3. On the other hand, the change of temperature over the window has not been taken into account for the thermal conductivity and using a somewhat smaller value therefore seems justified.

| **Material properties of Beryllium** | | |
|---|---|---|
| Thermal conductivity $\lambda$ | W/m/K | ~1500 |
| Thermal expansion coefficient $\alpha$ | 1/K | $1.3 \times 10^{-6}$ |
| Resistivity (77 K) | $\mu\Omega$cm | 0.07 |
| Resistivity (177 K) | $\mu\Omega$cm | 1.38 |

**Table 3:** Material Properties of beryllium at Liquid Nitrogen temperature

With these parameters the temperature rise across the window decreases to only 10 C° and the expansion to only 0.9 µm. The buckling height according to equ.( 10 ) is 0.35 mm. The actual temperature curves and numbers are shown in Figure 5. As stated before the crucial question remains whether these excellent beryllium properties can be achieved for the final window.

For the operation of the beryllium window with liquid nitrogen temperature, a number of other questions will have to be addressed which are beyond the scope of this paper but will have impact on the final decision to use these windows or not.

- The average power going into the cavity will not change by going to $LN_2$ operation. While the shunt impedance goes up (~Q), the peak power required for a given gradient goes down by the same amount, but the filling time increases by the same amount. So the average dissipated power stays constant. With the present parameters for the muon cooling channel [1] , this means approximately 6 kW of average power per meter of structure. This power is dissipated at $LN_2$. With a conversion factor of roughly 1:7 and a cooling channel length of at least 200 meters, this requires a 10 MW ac power $LN_2$ plant. The implications for construction and operation of such a plant can be found e.g. in

- In some of the references on Beryllium it is mentioned that Beryllium looses tensile strength if cooled to $LN_2$ temperature. Again this bulk property has to be tested.

- To date there is no engineering design available which would keep the beryllium foil under tension during rf operation.

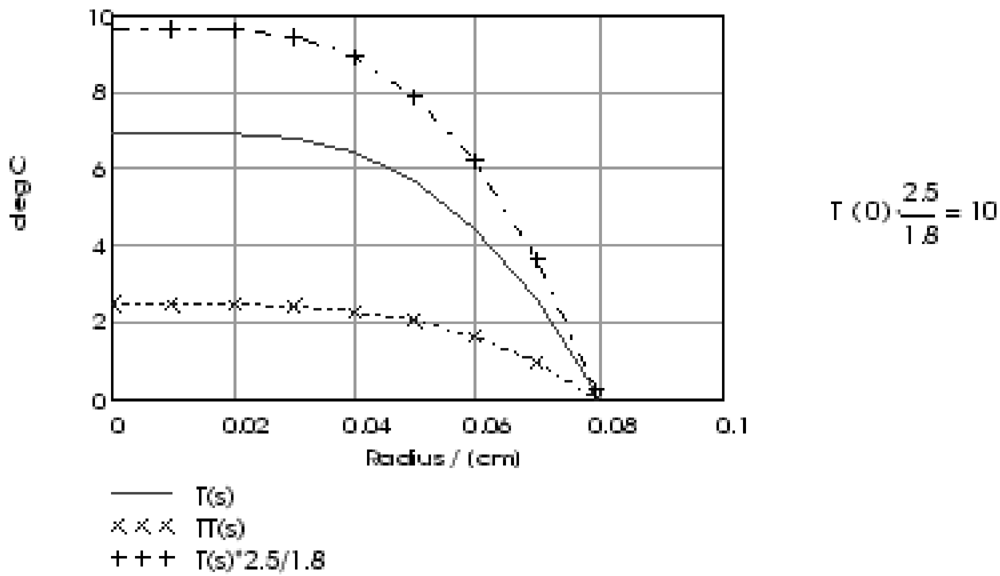



Figure 5: The picture shows the temperature distribution over the window. The dashed line (TT) is calculated not including the change in resistance with temperature. From the plot one can see that this effect rises the temperature by ~4 deg C. In addition, the dash-dotted line shows the difference between the pillbox cavity and the actual cavity as shown in Figure 1, which has lower shunt impedance.

## 7. Summary

The temperature gradient and the temperature distribution on a Beryllium window implemented in a 805 MHz pill box cavity has been investigated. While at room temperature the temperature rise over the 16 cm diameter window is almost 200 C°, at liquid nitrogen temperature the bulk properties in terms of ohmic losses seem very attractive. The possibility to reliably implement a Beryllium window overcoming such high temperature variations seems very difficult at room temperature which no technical solution available to date. At liquid nitrogen temperature on the other hand, bulk properties are questionable and implementation for high power operation at low temperature is just as difficult, although the stress is much less once the window is cooled down. Again no technical solution is available to date to keep the window stretched once it is cooled down.

## 8. Acknowledgements

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## 9. Bibliography

[1] *$\mu^+\mu^-$ Collider – A Feasibility Study* The Muon Collider Collaboration, published in Snowmass, 1996, BNL-52503, Fermilab Conf. 96/092, LBNL-38946. 2

[2] ***A Pi/2 Mode Interleaved Accelerating Structure for Muon Acceleration in a Cooling Channel***, A. Moretti et al, Linear Accelerator Conf. in Chicago, 1998, pp , 1998.

[3] ***Temperature Distribution Calculations on Beryllium Windows in RF Cavities for Muon Collider***, D. Li et al, Linear Accelerator Conf. in Chicago, 1998, , 1998.

[4] Bulk material Beryllium Properties, extracted from a suppliers brochure from Brush Wellman, Electrofusion Products, Fremont, CA, 1998.

[5] ***A Study of Refrigeration for Liquid Nitrogen-Cooled Power Transmission Cables***, R.C. Longsworth, K.F. Schoch, Advances in Cryogenic Eng., Vol 20, 1980.